AFCRC-TN-58-368
ASTIA Document No. AD160813

NEW YORK UNIVERSITY
Institute of Mathematical Sciences
Division of Electromagnetic Research

EM 116

RESEARCH REPORT No. EM-116

Surface Waves on a Right Angled Wedge

SAMUEL N. KARP and FRANK KARAL

CONTRACT No. AF 19(604)1717
AUGUST, 1958

EM-116 C.1

AFCRC-TN-58-368
ASTIA Document No. AD 160813

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The research reported in this document has been sponsored by the Electronics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command. The publication of this report does not necessarily constitute approval by the Air Force of the findings or conclusions contained herein.

Contract No. AF 19(604)1717

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Abstract

We investigate the fields produced by a line source located at the vertex of a right angled wedge, both of whose faces can sustain surface waves. The mathematical formulation of the problem involves the use of an impedance boundary condition. The impedance represents the effect of a corrugation of the surface, for example. It turns out that the radiated field is an elementary function of the observation angle. The surface wave amplitude tends to simple limiting values as its phase velocity tends to the velocity of propagation in free space, and also as the phase velocity is decreased without limit. The results are compared with similar results valid for a line source on an infinite plane. The method employed can be applied to treat wedges of more general angles.

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1. Introduction

In this paper we study the problem of the propagation of electromagnetic waves produced by a magnetic line dipole source located at the corner of a right angle wedge with an impedance boundary condition prescribed on the wedge surface. The boundary condition is such that surface waves are generated. The amplitudes of the surface waves generated are determined. A comparison is made between the magnitude of the surface waves for this problem and that of a magnetic line dipole source located on a plane surface characterized by the same impedance boundary condition. The comparison of the surface waves for the two different configurations is made on the assumption that the sources have the same source strength. Hence we can compare the change in the amplitudes of the surface waves as the wedge is straightened out into a plane, or, as the plane is bent into a right angled wedge. We also calculate the far field for both configurations.

The impedance boundary conditions prescribed in the problems considered are given by

$$(1.1) \qquad \frac{1}{r}\frac{\partial u}{\partial \theta} + \lambda u = 0 \quad , \quad \theta = 0$$

$$(1.2) \qquad \frac{1}{r}\frac{\partial u}{\partial \theta} - \lambda u = 0 \quad , \quad \theta = \alpha$$

where u is the z-component of the magnetic vector, r and θ are the usual polar coordinates, α is the wedge angle, and λ is a constant characteristic of the surface. The difference in sign appearing in the boundary conditions arises from the different orientation of the normals on the wedge faces. The value of λ is given by

$$(1.3) \qquad \lambda = + i\omega\epsilon \quad Z = + i\omega\epsilon\,(R-iX)$$

where ϵ is the permittivity of free space, ω is the angular frequency and Z, R and X are the impedance, resistance and reactance of the surface, respectively.

It is well known that for homogeneous media of large finite conductivity, R and X are positive in sign, small in magnitude and approximately equal. For illustrations of this boundary condition see Grunberg [10], Bazer and Karp [3], Fernando and Barlow [8] and Felsen [6]. For corrugated or dielectric-coated surfaces, although R and X are positive in sign, R is much smaller than X and hence Z is almost purely reactive. See Cullen [5], Felsen [7], Barlow and Cullen [1], Brick [4], Friedman and Williams [9] and Barlow and Karbowiak [2]. We note that when X is positive, equation (1.3) implies

(1.4) $\quad \mathrm{Re}\ \lambda > 0.$

This condition is employed in our mathematical work. The condition $R \ll X$ is of course the most important special case insofar as surface waves are concerned.

The problem we treat is not separable because of the mixed boundary condition.* This difficulty can, however, be overcome by introducing an auxiliary function which is a linear combination of the magnetic field and its cartesian derivatives. The auxiliary function is chosen in such a way that it satisfies the wave equation and simple homogeneous boundary conditions. Once the auxiliary function is obtained, the original field must be determined by solving an auxiliary partial differential equation. This idea is due to Stoker [17] and Lewy [13] who studied problems in water wave theory. It has since been employed by the authors (See Karp [12] and Karal and Karp [11]) for solving problems in diffraction theory.** A more general method of water wave theory (See Peters, [16]) has been adapted by Senior [15], who solves the two dimensional

---

* If the impedance varies as 1/r rather than being constant, the problem is separable. This is the problem treated by Felsen [7].

** In the present problem, the solution of the auxiliary partial differential equation involves integration in the Riemann surface of the multivalued auxiliary function. This complication is caused by condition (1.4).

scalar problem of a plane wave incident on a wedge of completely arbitrary angle, under certain impedance boundary conditions. Surface waves are not discussed in Senior's work since they are excluded by the boundary conditions.

In Section 2 we solve the problem of a magnetic line dipole located on a plane surface characterized by an impedance boundary condition. Although the problem is easily solvable by the use of Fourier transforms, a solution by the method of the present paper is given here, with a view to explaining the procedure in the case of a simple problem. The amplitudes of the surface waves generated are determined. In Section 3 we solve the problem of a magnetic line dipole source located at the tip of a right angle wedge characterized by the same impedance boundary condition. The amplitudes of the surface waves generated are also determined. This is a more difficult problem than that solved in Section 2, but the general method is the same . Section 3 also contains a comparison of the magnitudes of the surface waves for a wedge and a plane in the limiting cases of $\lambda \to 0$ and $\lambda \to \infty$ respectively, under the assumption that the source strengths in both cases are equal. In Section 4 we determine the far fields for both configurations. Appendix I contains a discussion and evaluation of several important double integrals. Appendix II discusses limiting expressions for certain important integrals that occur in the analysis.

2. Solution of the plane problem

In this section we study the electromagnetic field which arises from a magnetic line dipole lying on the surface of a plane characterized by an impedance boundary condition. Consider the plane surface $y = 0$ and suppose there is a magnetic line dipole located at the origin $x = 0$, $y = 0$. (See Figure 1.) In the region $y \geq 0$ we assume that we have free space. The boundary condition

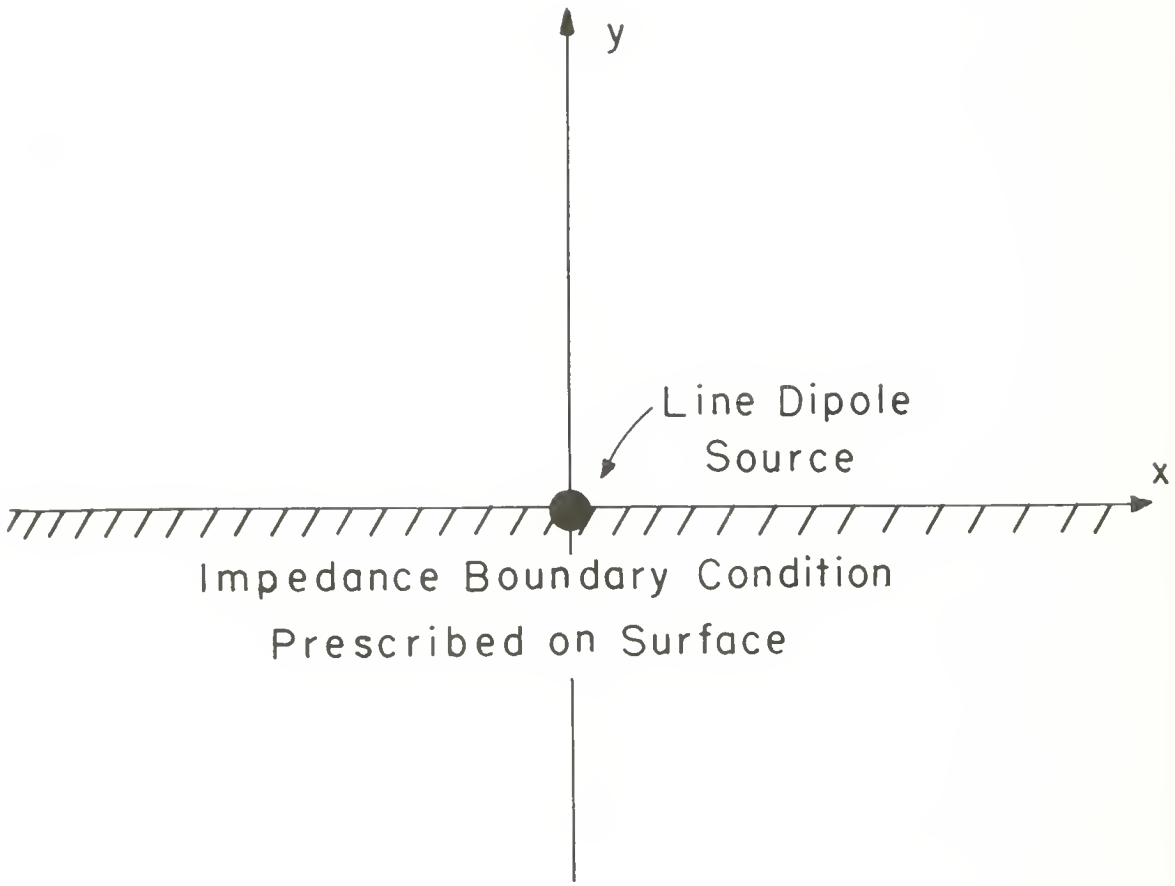
y
Line Dipole
Source
x
Impedance Boundary Condition
Prescribed on Surface

Figure 1

on the surface is given by the impedance condition

$$(2.1) \qquad \frac{\partial u}{\partial y} + \lambda u = 0 \quad , \quad y = 0$$

where u is the magnetic vector H_z and λ is a constant related to the conductivity of the material comprising the surface. We wish to solve the time reduced Maxwell's equations, subject to the prescribed conditions, and obtain the amplitudes of the surface waves propagated along the surface.

The time dependent form of Maxwell's equations is

$$(2.2) \qquad \begin{cases} \text{curl } \vec{H} = - i\omega\epsilon \vec{E} \\ \text{curl } \vec{E} = i\omega u \vec{H} \end{cases}$$

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic field intensities, and ϵ and u are the permittivity and magnetic permeability of free space. We assume the time dependence to be of the form $e^{-i\omega t}$. Because of the geometry, the field produced is independent of z and hence the field is completely determined by the value of H_z. We have

$$(2.3) \qquad H_x = H_y = E_z = 0$$

and

$$(2.4) \qquad \begin{cases} E_x = - \dfrac{1}{i\omega\epsilon} \dfrac{\partial H_z}{\partial y} \\ E_y = + \dfrac{1}{i\omega\epsilon} \dfrac{\partial H_z}{\partial x} \end{cases} .$$

The field component $H_z \equiv u$ satisfies the equation

$$(2.5) \qquad (\nabla^2 + k^2) u = - 4\pi\delta(x-0)\, \delta(y-0)$$

where ∇^2 is the rectangular Laplacian, k is the propagation constant of free space and δ is the Dirac delta function. Therefore, the mathematical problem reduces to that of solving the inhomogeneous wave equation (2.5) subject

to the mixed boundary condition given by (2.1). In addition to the prescribed boundary condition, we require that the far field be outgoing, and that the value of the secondary* magnetic field be finite everywhere.

Let us make the substitution

$$v = \frac{\partial u}{\partial y} + \lambda u \quad . \tag{2.6}$$

Then v satisfies the following equation

$$(\nabla^2 + k^2)v = 0 \quad , \quad x = y \neq 0 \quad , \tag{2.7}$$

subject to the simpler conditions

$$\begin{cases} (1)\ v = 0 \qquad , y = 0 \\ (2)\ \text{outward traveling waves at } \infty \quad . \end{cases} \tag{2.8}$$

If the problem for v can be solved, then we can obtain the solution for the problem involving u, since (2.6) can be integrated. One particular solution of (2.6) is

$$u_p(x,y) = e^{-\lambda y} \int_{-\infty}^{y} e^{+\lambda \eta}\, v(x, \eta)\, d\eta \quad . \tag{2.9}$$

It is important to point out that the function v does not have to be finite at the origin. Thus we should first introduce all radiating solutions, no matter how singular at the origin, that satisfy the appropriate boundary conditions for v. Out of this class of solutions we shall finally select only those that yield an everywhere finite value of the secondary magnetic field in the original physical problem involving u. Now, if we employ the method of separation of variables and use conditions (2.8), we find that the most general representation for u is of the form $\sum a_n H_n^{(1)}(kr)\sin n\theta$. Since v

---

* By secondary magnetic field we mean the total field u minus the field of the exciting line source. The primary field, or the field of the line source, is $i\pi H_o^{(1)}\left(k\sqrt{x^2+y^2}\right)$.

cannot be too singular, the only admissible function of this class is

$$(2.10) \qquad v = c_o H_1^{(1)}(kr) \sin\theta = -\frac{c_o}{k}\frac{\partial}{\partial y} H_o^{(1)}(kr).$$

The constant c_o has yet to be determined. This solution obviously satisfies the wave equation (2.7) and the boundary conditions (2.8). We substitute the second form of (2.10) into (2.9) and integrate by parts. Introducing a new constant c_1 for convenience, the particular solution (2.9) becomes

$$(2.11) \qquad u_p(x,y) = c_1\left[H_o^{(1)}(kr) - \lambda e^{-\lambda y}\int_{-\infty}^{y} e^{\lambda\eta}\, H_o^{(1)}(k\sqrt{x^2+\eta^2}\,)\,d\eta\right]$$

where $H_o^{(1)}(kr)$ is a Hankel function of the first kind of order zero, and c_1 (which is to be determined later) is a constant related to the source strength. Notice that in (2.9) and (2.11) we integrate below the boundary of the physical space. This is possible because the wave function v vanishes on the x axis and hence can be continued to negative values of y. The function u_p does not satisfy all the conditions of the problem, for it is not a wave function in regions including the positive y axis, since the x derivative is discontinuous at that line. This defect can be immediately remedied, as will be seen below, by adding to (2.11) another solution of the wave equation as follows:

$$(2.12) \qquad u(x,y) = c_1\left[H_o^{(1)}(kr) - \lambda e^{-\lambda y}\int_{-\infty}^{y} e^{\lambda\eta}\, H_o^{(1)}(kr)\,d\eta\right] + c_2 e^{-\lambda y + i\sqrt{k^2+\lambda^2}\,|x|}.$$

The introduction of the additional term with the unknown constant c_2 now makes it possible to satisfy the continuity condition involving $\frac{\partial u}{\partial x}$. We note that the form of the additional term is that of a surface wave. Hence, when we impose the continuity requirement on $\frac{\partial u}{\partial x}$ and evaluate c_2, we obtain the

amplitudes of the surface waves propagated away from the source. We now proceed to determine the value of c_2.

It follows at once from (2.12) that

$$(2.13)\qquad \frac{\partial u}{\partial x} \equiv u_x = c_1\left[\frac{\partial}{\partial x} H_o^{(1)}(kr) - \lambda e^{-\lambda y} \frac{\partial}{\partial x}\int_{-\infty}^{y} e^{\lambda\eta} H_o^{(1)}(kr)d\eta\right]$$

$$\pm c_2 i\sqrt{k^2+\lambda^2}\, e^{-\lambda y + i\sqrt{k^2+\lambda^2}\,|x|}$$

where the plus sign applies when $x > 0$, and the minus sign, when $x < 0$. Since

$$(2.14)\qquad \begin{cases} \dfrac{\partial}{\partial x} H_o^{(1)}(kr) = -kH_1^{(1)}(kr)\cos\theta \\ \dfrac{\partial}{\partial y} H_o^{(1)}(kr) = -kH_1^{(1)}(kr)\sin\theta \end{cases},$$

equation (2.13) can be written

$$(2.15)\qquad u_x(x,y) = -c_1 k\left[H_1^{(1)}(kr)\cos\theta - \lambda e^{-\lambda y}\int_{-\infty}^{y} e^{\lambda\eta} H_1^{(1)}(kr)\cos\theta\, d\eta\right]$$

$$\pm c_2 i\sqrt{k^2+\lambda^2}\, e^{-\lambda y + i\sqrt{k^2+\lambda^2}\,|x|} .$$

We require that u_x be continuous across the line $x = 0$. We can state this requirement as a jump condition as follows:

$$(2.16)\qquad \left[u_x\right] = \lim_{x\to 0}\left\{u_x(x>0,y) - u_x(x<0,y)\right\} = 0 .$$

Substituting (2.15) into the above jump condition, and noting that the first terms for u_x yield a zero contribution to the jump, we have

$$(2.17)\qquad \left[u_x\right] = \lim_{x\to 0}\left\{+2c_1 k\lambda e^{-\lambda y}\int_{-\infty}^{y} e^{\lambda\eta} H_1^{(1)}\left(k\sqrt{x^2+\eta^2}\right)\frac{x\,d\eta}{\sqrt{x^2+\eta^2}}\right.$$

$$\left. + 2c_2 i\sqrt{k^2+\lambda^2}\, e^{-\lambda y + i\sqrt{k^2+\lambda^2}\,|x|}\right\} .$$

The integral on the right, in the limit, is clearly zero except in the neighborhood of $\eta = 0$. Hence

$$(2.18)\qquad c_2 = \frac{+ic_1 k\lambda}{\sqrt{k^2+\lambda^2}} \operatorname*{Lim}_{x\to 0} \left\{ x \int_{-\epsilon}^{\epsilon} e^{\lambda\eta} H_1^{(1)}\left(k\sqrt{x^2+\eta^2}\right) \frac{d\eta}{\sqrt{x^2+\eta^2}} \right\}$$

where ϵ is small. Since both ϵ and x are small, the argument of H_1 is small. Using the well-known asymptotic formula for small argument, we find that the above simplifies to

$$(2.19)\qquad c_2 = \frac{ic_1 k\lambda}{\sqrt{k^2+\lambda^2}} \operatorname*{Lim}_{x\to 0} \left\{ x \int_{-\epsilon}^{\epsilon} (1) \left(\frac{-2i}{\pi k\sqrt{x^2+\eta^2}}\right) \frac{d\eta}{\sqrt{x^2+\eta^2}} \right\}$$

or

$$(2.20)\qquad c_2 = \frac{2\lambda c_1}{\pi\sqrt{k^2+\lambda^2}} \operatorname*{Lim}_{x\to 0} \int_{-\epsilon}^{\epsilon} \frac{x}{x^2+\eta^2}\, d\eta \ .$$

The limit occurring in (2.20) is easily evaluated by making the substitution $\eta = xt$. Then we find that

$$(2.21)\qquad c_2 = \frac{+\,2\,\lambda c_1}{\pi\sqrt{k^2+\lambda^2}} \int_{-\infty}^{\infty} \frac{dt}{1+t^2} = \frac{2\lambda}{\sqrt{k^2+\lambda^2}}\, c_1 \ .$$

Therefore

$$(2.22)\qquad u(x,y) = c_1 \left[H_o^{(1)}(kr) - \lambda e^{-\lambda y} \int_{-\infty}^{y} e^{\lambda\eta} H_o^{(1)}(kr)\, d\eta + \frac{2\lambda}{\sqrt{k^2+\lambda^2}}\, e^{-\lambda y + i\sqrt{k^2+\lambda^2}|x|} \right].$$

The first term corresponds to the source, the second term represents the secondary contribution to the cylindrically radiated field,* and the last

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* It is easily shown that this integral which can be interpreted as an infinite distribution of line sources, does not contain a surface wave term. In fact a finite distribution of line sources is a cylindrically radiating function, and although our integral is an infinite distribution of line sources, the exponential decay factor makes it behave like a finite one.

term is the desired surface wave. The constant c_1 is related to the source strength and can be determined in the following simple way. The solution of (2.5) for a simple point source located at the origin is given by the well-known Green's function

$$G = i\pi H_o^{(1)}(kr) \quad . \tag{2.23}$$

Hence the proper normalization for a point source is obtained, provided that we let

$$c_1 = i\pi \quad . \tag{2.24}$$

We note that the amplitude of the surface wave is given by $\left| 2\pi \lambda / \sqrt{\lambda^2+k^2} \right|$. When λ/k is small the amplitude of the surface wave is $|2\pi \lambda/k|$, and when λ/k is large the amplitude tends to $|2\pi|$.

3. Solution of the right angled wedge problem

We now study the electromagnetic field which arises from a magnetic line dipole located at the tip of a right angled wedge whose surface is characterized by an impedance boundary condition. Consider the right angled wedge $y = 0$, $x > 0$ and $x = 0$, $y < 0$, and suppose there is a magnetic line dipole located at the origin $x = 0$, $y = 0$. (See Figure 2.) The boundary conditions on the surface are given by

$$\begin{cases} \dfrac{\partial u}{\partial y} + \lambda u = 0 & y = 0,\ x > 0 \\ \dfrac{\partial u}{\partial x} - \lambda u = 0 & x = 0,\ y < 0 \end{cases} \tag{3.1}$$

where u is the magnetic vector and λ is a parameter related to the impedance. (See Introduction.) We emphasize that Re $\lambda > 0$. We wish to solve the time-reduced Maxwell's equations and obtain the amplitude of the surface waves.

The field component $H_z = u$ satisfies the equation

$$(\nabla^2+k^2)u = - 4\pi\delta(x-0)\ \delta(y-0) \quad . \tag{3.2}$$

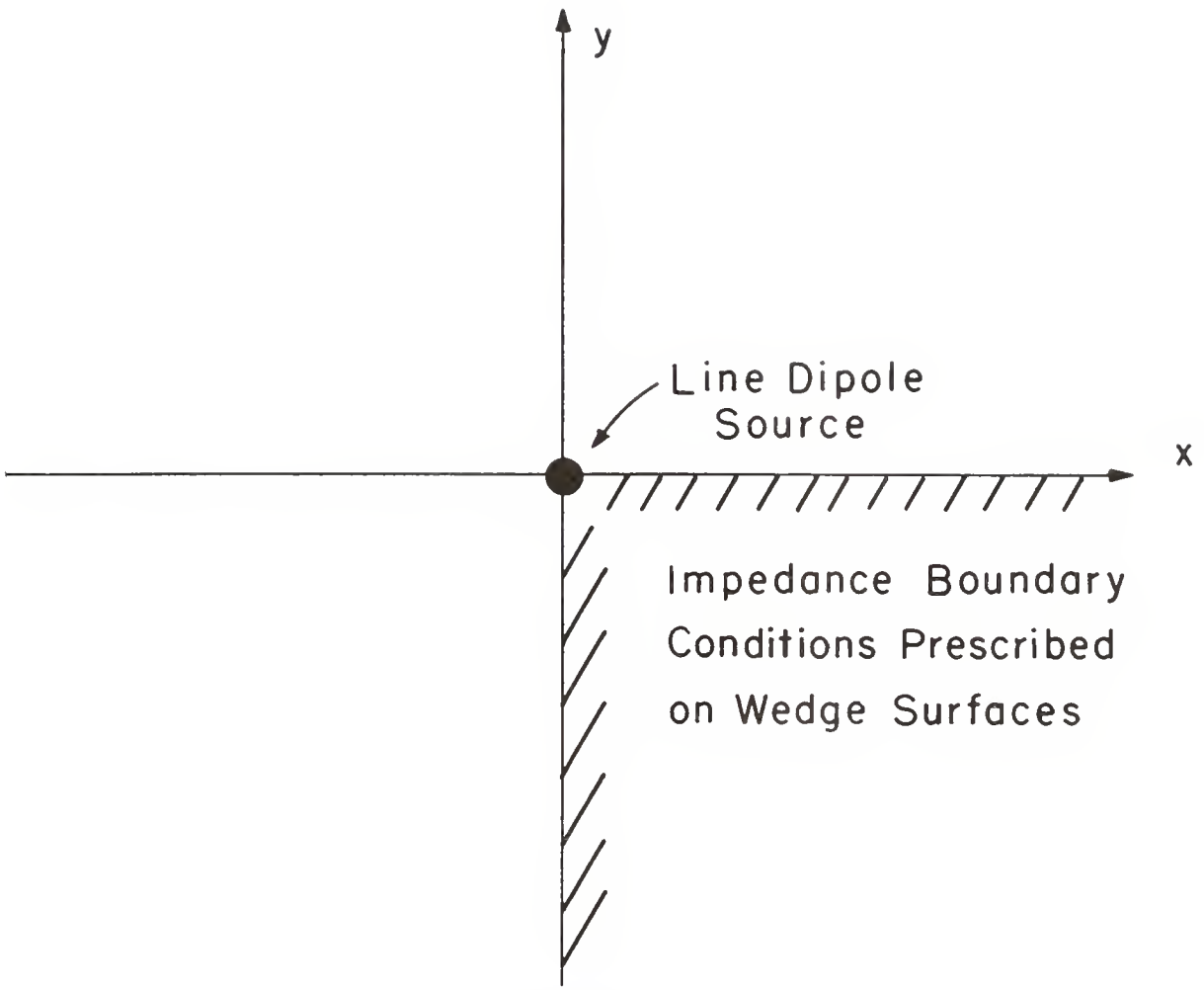
y
Line Dipole
Source
x
Impedance Boundary
Conditions Prescribed
on Wedge Surfaces

Figure 2

Let us make the substitution

$$(3.3) \qquad v = (\frac{\partial}{\partial y} + \lambda)U$$

where

$$(3.4) \qquad U = (\frac{\partial}{\partial x} - \lambda)u \ .$$

Then v satisfies the following equation

$$(3.5) \quad (\nabla^2+k^2)v = 0 \ , \quad x^2+y^2 \neq 0, \ 0 \leq \theta \leq 3\pi/2 \ ,$$

subject to the simple conditions

$$(3.6) \qquad \begin{cases} (1)\ v = 0 & y = 0, \quad x > 0 \\ (2)\ v = 0 & x = 0, \quad y < 0 \\ (3)\ \text{Outgoing waves at infinity.} \end{cases}$$

Conditions (3.6) can only be satisfied by a superposition of solutions of the form $H^{(1)}_{2n/3}(kr)\sin 2n/3\ \theta$, where n is an integer. We shall show that the only admissible solution for v has the form

$$(3.7) \qquad v = c_o H_2^{(1)}(kr)\sin 2\theta + c_1 H_{4/3}^{(1)}(kr)\sin \tfrac{4}{3}\theta + c_2 H_{2/3}^{(1)}(kr)\sin \tfrac{2}{3}\theta \ ,$$

where the various terms of (3.7) correspond to the values n = 3,2,1 respectively. The reason for this choice of values of n is that a double integration* must be performed upon v in order to yield u. Then the term H_2 leads to the desired source term H_o. The terms $H_{4/3}$, and $H_{2/3}$, which correspond to n=2 and n=1 respectively, lead to finite values at the origin after integration. Higher values of n would, however, lead to infinite values of u at the origin, and such values are therefore excluded. We note that an important simplification can be made at this stage. Observe first that u is clearly symmetric about

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* Note that (3.3) and (3.4) have the symbolic solution $u = (\frac{\partial}{\partial x} - \lambda)^{-1}(\frac{\partial}{\partial x} + \lambda)^{-1}v$. Hence a double integration is involved.

the line $y = -x$ so that $\frac{\partial u}{\partial n} = 0$ there. Next, if we combine (3.3) and (3.4), we see that $v = (\frac{\partial}{\partial x} - \lambda)(\frac{\partial}{\partial y} + \lambda)u$. If we introduce the derivatives $\frac{\partial}{\partial s}$ and $\frac{\partial}{\partial n}$, tangential and normal to the line $y = -x$, we find that $(\frac{\partial}{\partial x} - \lambda)(\frac{\partial}{\partial y} + \lambda)$ is a quadratic function in $\frac{\partial}{\partial s}$ alone. Hence we have $\frac{\partial v}{\partial n} = 0$ on the line $y = -x$ or $\frac{\partial v}{\partial \theta} = 0$ at $\theta = \frac{3\pi}{4}$. By applying this symmetry condition we find at once that

$$c_1 \equiv 0.$$

Consequently (3.7) contains only two terms involving the constants c_o and c_2.

The expression (3.7) for v is a wave function, defined by means of (3.5) in the angular section $0 \leq \theta \leq \frac{3\pi}{2}$. We note, however, that we can extend the range of v to the 4th quadrant. Since v vanishes on the positive x-axis, the possibility of the extension follows from the reflection principle. We carry out the extension by using (3.7) and permitting θ to range over the enlarged angular section $-\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$. Using the extended definition of v, we construct the expression

$$(3.8) \qquad e^{-\lambda y} \int_{\infty}^{y} e^{\lambda \eta} v(x, \eta)\, d\eta$$

which is easily seen to be a particular solution of (3.3). Since v is regular in the extended range $-\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$, we may differentiate (3.8) under the integral sign in this range and also integrate by parts. We can then easily verify that (3.8) is a wave function except across the line $x = 0$. Since we are integrating the inhomogeneous equation (3.3), we can add a complementary function (indicated in curly brackets in (3.9) below) to the expression in (3.8) and obtain a representation of the wave function U of (3.3) and (3.4) in the form

$$U(x,y) = e^{-\lambda y}\left[\int_{-\infty}^{y} e^{\lambda\eta}\left\{c_o H_2^{(1)}(kr)\sin 2\theta + c_2 H_{2/3}(kr)\sin\frac{2}{3}\theta\right\} d\eta\right]$$

(3.9)

$$+\left\{\begin{array}{ll} c_4 e^{-\lambda y + i\sqrt{k^2+\lambda^2}\,x}, & x > 0 \\ 0, & x < 0 \end{array}\right\} .$$

In the integrand of this expression $r = \sqrt{x^2 + \eta^2}$, $\theta = \arctan\frac{\eta}{x}$, and $-\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$. Except across the line $x = 0$, the term in curly brackets is a wave function, as is the term in square brackets. But, as we shall see, each term separately is discontinuous together with its derivative, at $x = 0$. Nevertheless, U is regular across the line $x = 0$, $y > 0$, and from this fact we can derive two relations between the constants c_o, c_2, and c_4. These relations are obtained by calculating the discontinuity of U and U_x across the positive y axis, and setting the discontinuities equal to zero.

Before calculating the discontinuities, it is convenient to transform the expressions for U. By using the relations

(3.10)

$$\left\{\begin{array}{l} H_{\nu-1}^{(1)}(kr)\sin(\nu-1)\theta = \frac{1}{k}\left[\frac{\partial}{\partial y}\left\{H_\nu^{(1)}(kr)\cos\nu\theta\right\} - \frac{\partial}{\partial x}\left\{H_\nu^{(1)}(kr)\sin\nu\theta\right\}\right] \\ H_{\nu+1}^{(1)}(kr)\sin(\nu+1)\theta = \frac{-1}{k}\left[\frac{\partial}{\partial y}\left\{H_\nu^{(1)}(kr)\cos\nu\theta\right\} + \frac{\partial}{\partial x}\left\{H_\nu^{(1)}(kr)\sin\nu\theta\right\}\right] \end{array}\right.$$

and integrating by parts, (3.9) becomes

$$U(x,y) = c_o\left\{-\frac{2}{k}H_1^{(1)}(kr)\cos\theta + \frac{2\lambda}{k}e^{-\lambda y}\int_{\infty}^{y} e^{\lambda\eta}H_1^{(1)}(kr)\cos\theta\, d\eta\right\} \tag{3.11}$$

$$+ c_2\left\{e^{-\lambda y}\int_{-\infty}^{y} e^{\lambda\eta}\, H_{2/3}^{(1)}(kr)\sin\frac{2\theta}{3}\, d\eta\right\}$$

$$+\begin{cases} c_4 e^{-\lambda y + i\sqrt{k^2+\lambda^2}\, x} & -\frac{\pi}{2} \le \arctan\frac{y}{x} < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le \arctan\frac{y}{x} \le \frac{3\pi}{2} \end{cases} .$$

Note that here $\theta = \theta(x, \eta)$, where $-\frac{\pi}{2} < \theta = \arctan\frac{\eta}{x} < \frac{3\pi}{2}$.

Now $U(x,y)$ and its derivatives are continuous when considered as functions of y. However, as noted earlier, the individual terms of $U(x,y)$ and $U_x(x,y)$ are not continuous functions of x across the line $x = 0$, $y > 0$, since, for example, for a fixed negative η, the integrands have different values according as x is positive or negative. Hence, for $U(x,y)$ to be a wave function, we must require that the following jump conditions be satisfied:

$$(3.12)\qquad \begin{cases} (I) & \left[U(0^+,y) - U(0^-,y)\right] = 0, \quad y > 0 \\ (II) & \left[U_x(0^+,y) - U_x(0^-,y)\right] = 0, \quad y > 0. \end{cases}$$

The first jump condition yields

$$(3.13)\quad c_o\frac{4\lambda}{k}\lim_{x\to 0}\int_{-\infty}^{0} e^{\lambda\eta}\, H_1^{(1)}(k|\eta|)\,\frac{x}{\sqrt{x^2+\eta^2}}\, d\eta - c_2\frac{\sqrt{3}}{2}\int_{-\infty}^{0} e^{\lambda\eta} H_{2/3}^{(1)}(k|\eta|)d\eta + c_4 = 0.$$

If we let

$$(3.14)\qquad I_\nu(k,\lambda) = \int_0^\infty e^{-\lambda\eta} H_\nu^{(1)}(k|\eta|)d\eta$$

the first jump condition reduces to

$$(3.15)\qquad -\frac{\sqrt{3}}{2}\, c_2\left[I_{2/3}\right] + c_4 = \frac{4\lambda i}{k^2}\, c_o .$$

We note that

$$(3.16)\qquad I_\nu(k,\lambda) = \left(\frac{i}{\sin \nu\pi}\right) \frac{e^{-i\nu\frac{\pi}{2}}}{k^\nu \sqrt{\lambda^2+k^2}} \left[e^{-i\nu\frac{\pi}{2}} \left(\sqrt{\lambda^2+k^2} - \lambda \right)^\nu - e^{i\nu\frac{\pi}{2}} \left(\sqrt{\lambda^2+k^2} + \lambda \right)^\nu \right]$$

where $-1 > \mathrm{Re}\ \nu < +1$. (See [14].)

Consider next the second jump condition. We first calculate $U_x(x,y)$ and find, after integrating by parts, that

$$\begin{aligned}(3.17)\qquad U_x(x,y) = c_o \Bigg\{ & -\frac{2}{k}\frac{\partial}{\partial x}\left[H_1^{(1)}(kr)\cos\theta\right] - \frac{2\lambda}{k} H_1^{(1)}(kr)\sin\theta \\ & + \frac{2\lambda^2}{k} e^{-\lambda y} \int_{-\infty}^{y} e^{\lambda\eta} H_1^{(1)}(kr)\sin\theta\, d\eta + 2\lambda e^{-\lambda y} \int_{-\infty}^{y} e^{\lambda\eta} H_o^{(1)}(kr)\, d\eta \Bigg\} \\ & + c_2 \Bigg\{ H_{2/3}^{(1)}(kr)\cos\frac{2\theta}{3} - \lambda e^{-\lambda y} \int_{-\infty}^{y} e^{\lambda\eta} H_{2/3}^{(1)}(kr)\cos\frac{2\theta}{3}\, d\eta \\ & \qquad - k e^{-\lambda y} e^{i\frac{\pi}{3}} \int_{-\infty}^{y} e^{\lambda\eta} H_{1/3}^{(1)}(kr)\sin\frac{\theta}{3}\, d\eta \Bigg\} \\ & + \begin{cases} i\sqrt{k^2+\lambda^2}\, c_4 e^{-\lambda y + i\sqrt{k^2+\lambda^2}\, x}, & -\frac{\pi}{2} < \arctan\frac{y}{x} < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < \arctan\frac{y}{x} < \frac{3\pi}{2} \end{cases} .\end{aligned}$$

Substituting into the second jump condition we find

$$(3.18)\qquad + c_2\left[e^{i\frac{\pi}{3}} I_{1/3} - \frac{\lambda}{k} I_{2/3}\right] + c_4\left[\frac{2}{3} i \frac{\sqrt{k^2+\lambda^2}}{k}\right] = 0 .$$

We now have two equations which relate the three unknown constants c_o, c_2 and c_4. By requiring that $U(x,y)$ and $U_x(x,y)$ be continuous, we have ensured that $U(x,y)$ is a wave function everywhere. We now proceed to the determination of $u(x,y)$.

The function $U(x,y)$, as given by (3.9), and subject to the conditions

(3.15) and (3.18), is a wave function defined in the angular sector $-\frac{\pi}{2} \leq \arctan\frac{y}{x} \leq \frac{3\pi}{2}$. In order to obtain u(x,y) from U(x,y) by integrating (3.4), we first extend this region of definition to the angular sector $-\frac{\pi}{2} \leq \arctan\frac{y}{x} \leq 2\pi$. The Schwartz reflection principle shows that this is possible, since U(x,y) vanishes at x=0, y < 0. If we perform this extension from the third to the fourth quadrant, then we note that if $|y| \neq 0$, a ξ (or x) integration in (3.19) can proceed continuously without encountering a barrier. Thus a particular solution of (3.4) is

$$(3.19) \qquad -e^{\lambda x}\int_{x}^{\infty} e^{-\lambda\xi}U(\xi,y)d\xi$$

and the regularity of U outside the positive x-axis can be used to show by direct differentiation that u (regardless of the value of the c's implied in U), is a wave function unless y = 0, x < 0. In the subsequent analysis we frequently make use of polar coordinates because the integrands are multivalued functions of the cartesian coordinates. Since there is an overlap of the regions of definition of the integrands (in the fourth quadrant), care must be exercised in performing the integrations. With the extended definition of U(x,y) we can now obtain a well-defined particular solution of equation (3.4) by substituting the individual terms (3.9) into (3.19). We add a complementary solution of (3.4), (given in curly brackets in 3.20 below) carry integrations out explicitly where practicable and convenient, and obtain, for the desired function u, the following expression:

$$(3.20) \quad u(x,y) = \frac{2}{k^2}c_o H_o^{(1)}(kr) - \frac{2\lambda}{k^2}c_o e^{\lambda x}\int_{x}^{\infty} e^{-\lambda\xi}H_o^{(1)}(kr)d\xi$$
$$- \frac{2\lambda}{k}c_o\, e^{-\lambda(y-x)}\int_{x}^{\infty} e^{-\lambda\xi}d\xi \int_{-\infty}^{y} e^{\lambda\eta}\, H_1^{(1)}(kr)\cos\theta\, d\eta$$

$$- c_2 e^{-\lambda(y-x)} \int_x^{\infty} e^{-\lambda\xi} d\xi \int_{-\infty}^{y} e^{\lambda} H_{2/3}^{(1)}(kr)\sin\frac{2\theta}{3}\, d\eta$$

$$+ \frac{c_4 e^{-\lambda y + i\sqrt{k^2+\lambda^2}\, x}}{-\lambda + i\sqrt{k^2+\lambda^2}} h(x)h(y) + \frac{c_4 e^{-\lambda y+\lambda x}}{-\lambda+i\sqrt{k^2+\lambda^2}} h(-x)h(y)$$

$$+ c_3 e^{\lambda x - i\sqrt{k^2+\lambda^2}\, y} h(-y) \quad .$$

In this expression the function $h(x)$ is equal to unity for $x > 0$ and to zero for $x < 0$, and $h(y)$, $h(-x)$, $h(-y)$ are defined accordingly. Certain integrations have been performed by parts, on the model of the calculations performed earlier, in the transition from (3.9) to (3.11). A new constant c_3 has been introduced.

We shall now proceed to deduce two additional relations between the constants c_o, c_2, c_3 and c_4. These relations follow from the fact that $U(x,y)$ and its derivative must be continuous across the line $y = 0$, $x < 0$. We can express the continuity conditions as jump conditions as follows:

$$(3.21) \qquad \begin{cases} (III) & \left[u(x,0^+) - u(x,0^-)\right] = 0, \quad x < 0 \\ (IV) & \left[u_y(x,0^+) - u_y(x,0^-)\right] = 0, \quad x < 0. \end{cases}$$

Jump condition III yields*

$$(3.22) \qquad \left[- \frac{2\lambda}{k} c_o e^{\lambda x} \int_0^{\infty} \int_{-\frac{\pi}{2}}^{0} e^{\lambda r(\cos\theta - \sin\theta)} H_1^{(1)}(kr)\cos\theta\, r\, dr d\theta \right.$$

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* In (3.22) we omit the first two terms of u, as given by (3.20), since the jumps of these terms are obviously zero.

$$- c_2 e^{\lambda x} \int_0^{\infty} \int_{\frac{\pi}{2}}^{0} e^{-\lambda r(\cos\theta - \sin\theta)} H_{2/3}^{(1)}(kr) \sin\frac{2\theta}{3}\, r\,dr\,d\theta + \frac{c_4 e^{\lambda x}}{-\lambda + i\sqrt{k^2+\lambda^2}} \Bigg]$$

$$- \Bigg[- \frac{2\lambda}{k} c_o e^{\lambda x} \int_0^{\infty} \int_{\frac{3\pi}{2}}^{2\pi} e^{-\lambda r(\cos\theta - \sin\theta)} H_1^{(1)}(kr) \cos\theta\, r\,dr\,d\theta$$

$$- c_2 e^{\lambda x} \int_0^{\infty} \int_{\frac{3\pi}{2}}^{2\pi} e^{-\lambda r(\cos\theta - \sin\theta)} H_{2/3}^{(1)}(kr) \sin\frac{2\theta}{3}\, r\,dr\,d\theta + c_3 e^{\lambda x} \Bigg] = 0$$

or

$$(3.23) \quad - \frac{2\lambda}{k} c_o H - c_2 J + \frac{c_4}{-\lambda + i\sqrt{k^2+\lambda^2}} - c_3 = 0 \,.$$

In the above expressions

$$(3.24) \quad H = \int_0^{\infty} \int_{-\frac{\pi}{2}}^{0} e^{-\lambda r(\cos\theta - \sin\theta)} H_1^{(1)}(kr) \cos\theta\, r\,dr\,d\theta$$

$$- \int_0^{\infty} \int_{\frac{3\pi}{2}}^{2\pi} e^{-\lambda r(\cos\theta - \sin\theta)} H_1^{(1)}(kr) \cos\theta\, r\,dr\,d\theta$$

$$(3.25) \quad J = \int_0^{\infty} \int_{\frac{\pi}{2}}^{0} e^{-\lambda r(\cos\theta - \sin\theta)} H_{2/3}^{(1)}(kr) \sin\frac{2\theta}{3}\, r\,dr\,d\theta$$

$$- \int_0^{\infty} \int_{\frac{3\pi}{2}}^{2\pi} e^{-\lambda r(\cos\theta - \sin\theta)} H_{2/3}^{(1)}(kr) \sin\frac{2\theta}{3}\, r\,dr\,d\theta$$

and I_ν is defined in (3.16). Using the fact that the jumps H and J are zero (See Appendix I), we have

$$(3.26) \qquad -c_3 + \frac{c_4}{-\lambda+i\sqrt{k^2+\lambda^2}} = 0.$$

Consider next the fourth jump condition. First, however, we need $u_y(x,y)$. We find that u_y can be written in the following form if we use the identities (3.10) and integrate by parts:

$$\begin{aligned}(3.27) \qquad u_y(x,y) = &-\frac{2}{k}c_o H_1^{(1)}(kr)\sin\theta + \frac{2\lambda}{k}c_o e^{\lambda x}\int_x^\infty e^{-\lambda\xi}H_1^{(1)}(kr)\sin\theta\,d\xi \\ &+ \frac{2\lambda^2}{k}c_o e^{-\lambda(y-x)}\int_x^\infty e^{-\lambda\xi}\int_{-\infty}^y e^{\lambda\eta}H_1^{(1)}(kr)\cos\theta\,d\eta\,d\xi \\ &- \frac{2\lambda}{k}c_o e^{\lambda x}\int_x^\infty e^{-\lambda\xi}H_1^{(1)}(kr)\cos\theta\,d\xi - \frac{2}{k}c_1 H_{1/3}(kr)\sin\frac{\theta}{3} \\ &+ \lambda c_2 e^{-\lambda(y-x)}\int_x^\infty e^{-\lambda\xi}\int_{-\infty}^y e^{\lambda\eta}H_{2/3}^{(1)}(kr)\sin\frac{2\theta}{3}\,d\eta\,d\xi \\ &- c_2 e^{\lambda x}\int_x^\infty e^{-\lambda\xi}H_{2/3}^{(1)}(kr)\sin\frac{2\epsilon}{3}\,d\xi \\ &+ \frac{-\lambda c_4 e^{-\lambda y+i\sqrt{k^2+\lambda^2}\,x}\cdot h(x)h(y)}{-\lambda+i\sqrt{k^2+\lambda^2}} - i\sqrt{k^2+\lambda^2}\,c_3 e^{\lambda x-i\sqrt{k^2+\lambda^2}\,y}h(-x)h(-y) \\ &+ \frac{-\lambda c_4 e^{-\lambda y+\lambda x}\cdot h(-x)h(y)}{-\lambda+i\sqrt{k^2+\lambda^2}}.\end{aligned}$$

If we now substitute into the fourth jump condition and collect terms we have, after some reduction,

$$(3.28) \qquad -c_2\frac{\sqrt{3}}{2}\frac{1}{k}I_{2/3} + i\sqrt{1+\frac{\lambda^2}{k^2}}\,c_3 - \frac{1}{k}\frac{\frac{\lambda}{k}c_4}{-\frac{\lambda}{k}+i\sqrt{1+\frac{\lambda^2}{k^2}}} = +\frac{4\lambda i c_o}{k^3}.$$

We now have four linear algebraic equations given by (3.15), (3.18), (3.26) and (3.28) for the four unknowns c_o, c_2, c_3 and c_4 respectively. The constant

c_o, however, is related to the source strength and can be determined by proper normalization. In order to have a simple point source as defined in (2.24), we find that we must set

$$(3.29) \qquad c_o = \frac{i\pi k^2}{2} \quad .$$

This leaves us with four equations and three unknowns. We note, however, that equations (3.15) and (3.28) are identical. Hence we really have three equations involving three unknowns. We summarize them as follows:

$$(3.30) \qquad c_3 = \frac{c_4}{i\sqrt{k^2+\lambda^2} - \lambda}$$

$$(3.31) \qquad - c_2 \frac{\sqrt{3}}{2} I_{2/3} + (i\sqrt{\lambda^2+k^2} - \lambda)c_3 = \frac{4\lambda i c_o}{k^2}$$

$$(3.32) \qquad c_2\left[e^{i\frac{\pi}{3}} I_{1/3} - \frac{\lambda}{k} I_{2/3}\right] + \frac{2i}{3} c_3 \frac{\sqrt{k^2+\lambda^2}}{k} (i\sqrt{\lambda^2+k^2} - \lambda) = 0 \ .$$

Reference to (3.20) shows that the quantities on the left and right hand sides of (3.30) are the surface wave amplitudes on the wedge faces. Hence equation (3.30) means that the amplitudes of the surface waves on both surfaces are equal, which was to be expected due to the symmetry of the problem. Solving (3.31) and (3.32) we find that

$$(3.33) \qquad c_2 = \frac{4\sqrt{3}\,\pi i}{3} \frac{\lambda\sqrt{k^2+\lambda^2}}{\left\{\left(i\sqrt{k^2+\lambda^2} - \sqrt{3}\,\lambda\right) I_{2/3} + \sqrt{3}\,ke^{i\frac{\pi}{3}} I_{1/3}\right\}}$$

$$(3.34) \qquad c_3 = -2\sqrt{3}\,\pi \frac{\left\{ke^{i\frac{\pi}{3}} I_{1/3} - \lambda I_{2/3}\right\}}{\left\{\left(i\sqrt{k^2+\lambda^2} - \sqrt{3}\,\lambda\right) I_{2/3} + \sqrt{3}\,ke^{i\frac{\pi}{3}} I_{1/3}\right\}} \frac{\lambda}{(i\sqrt{k^2+\lambda^2}-\lambda)} \ .$$

Here I_ν is the elementary function given in equation (3.16). The amplitude

of the surface wave in the third quadrant is given by c_3; the amplitude of the surface wave in the first quadrant is given by $c_4 / \left(i \sqrt{k^2+\lambda^2} - \lambda \right)$.

If $\frac{\lambda}{k}$ is small, the above results simplify. Keeping terms of the same order we find that

$$(3.35) \qquad c_2 = \frac{i\pi}{2} \left(1 - \frac{i}{\sqrt{3}}\right) \lambda k$$

$$(3.36) \qquad c_3 = i\pi \frac{\lambda}{k}$$

$$(3.37) \qquad c_4 = - \pi\lambda$$

where we have used the following approximate expressions for $I_{1/3}$ and $I_{2/3}$:

$$(3.38) \qquad I_{1/3} \approx \frac{1}{k}\left(1 - \frac{i}{\sqrt{3}}\right)$$

$$(3.39) \qquad I_{2/3} \approx \frac{1}{k}(1 - i\sqrt{3}).$$

(See Appendix II.) We note that the amplitude of the surface wave is given by $|\pi \frac{\lambda}{k}|$. The ratio of the amplitude of the surface wave on the wedge surface to that on the plane surface is given by $\frac{1}{2}$ provided $\frac{\lambda}{k}$ is small.

We next treat the case in which $\frac{\lambda}{k}$ is large. Keeping terms of the same order we find that

$$(3.40) \qquad c_2 = \frac{2\pi}{\sqrt{3}\ \sqrt[3]{4}} \ \frac{k^2}{(1 - i/\sqrt{3})} \ \left(\frac{\lambda}{k}\right)^{4/3}$$

$$(3.41) \qquad c_3 = \frac{2\pi}{(1-i)(1- i/\sqrt{3})}$$

$$(3.42) \qquad c_4 = - \frac{2\pi\lambda}{(1- i/\sqrt{3})} .$$

We have used the following approximate expressions which are derived in Appendix II:

$$(3.43) \qquad I_{1/3} \approx -\, i\, \frac{2}{\sqrt{3}} \left(\frac{2}{k}\right)^{1/3} \frac{1}{\lambda^{2/3}}$$

$$(3.44) \qquad I_{2/3} \approx -\, i\, \frac{2}{\sqrt{3}} \left(\frac{2}{k}\right)^{2/3} \frac{1}{\lambda^{1/3}} \quad .$$

We note that the amplitude of the surface wave is given by $\left|\pi \sqrt{\frac{3}{2}}\right|$. The ratio of the amplitude of the surface wave on the wedge surface to that on the plane surface is given by $\frac{1}{2}\sqrt{\frac{3}{2}}$ provided that $\frac{\lambda}{k}$ is large.

4. Determination of the radiated far fields.

We first determine the radiated far field expression for the case of a magnetic-line dipole source located on a plane surface. From (2.6) and (2.10) we have

$$(4.1) \qquad v = \left(\frac{\partial}{\partial y} + \lambda\right) u$$

and

$$(4.2) \qquad v = c_o H_1^{(1)}(kr) \sin\theta$$

where

$$(4.3) \qquad c_o = -kc_1 = -i\pi k \ .$$

When kr is large (4.2) becomes

$$(4.4) \qquad v = -\, i\pi k \sqrt{\frac{2}{\pi kr}}\, e^{i\left(kr - \frac{3\pi}{4}\right)} \sin\theta \ .$$

We expect the far field of the z-component of the magnetic field vector u to have a similar form. Hence

$$(4.5) \qquad u = \frac{\ell(\theta)}{\sqrt{r}}\, e^{ikr}$$

where $\ell(\theta)$ is unknown.

Now

$$(4.6) \qquad \frac{\partial}{\partial x} = \cos\theta \frac{\partial}{\partial r} - \frac{1}{r} \sin\theta \frac{\partial}{\partial \theta}$$

$$(4.7) \qquad \frac{\partial}{\partial y} = \sin\theta \frac{\partial}{\partial r} + \frac{1}{r} \cos\theta \frac{\partial}{\partial \theta} .$$

Substituting into (4.1) and keeping terms of order $r^{-1/2}$, we obtain

$$(4.8) \qquad \ell(\theta) = \sqrt{2\pi k} \frac{\sin\theta \; e^{-i\frac{5\pi}{4}}}{(ik \sin\theta + \lambda)} .$$

Hence

$$(4.9) \qquad u = \sqrt{\frac{2\pi}{kr}} \frac{ik \sin\theta}{(ik \sin\theta + \lambda)} e^{ik(r - \frac{7\pi}{4})} .$$

When $\frac{\lambda}{k}$ is small the far field is given by

$$(4.10) \qquad u = \sqrt{\frac{2\pi}{kr}} e^{ik(r - \frac{7\pi}{4})} (1 - \lambda/ik \sin\theta) \cong i\pi H_o(kr)(1 - \frac{\lambda}{ik \sin\theta})$$

provided $\theta \neq 0, \pi$. The first factor is the field of the incident source.

When $\frac{\lambda}{k}$ is large the far field is given by

$$(4.11) \qquad u = \sqrt{\frac{2\pi}{kr}} (\frac{k}{\lambda}) \sin\theta \; e^{ik(r - \frac{5\pi}{4})} .$$

We next determine the radiated far field expression for the case of a magnetic line dipole located at the corner of a right angled wedge. From (3.3), (3.4) and (3.9) we have

$$(4.12) \qquad v = (\frac{\partial}{\partial y} + \lambda)(\frac{\partial}{\partial x} - \lambda)u$$

and

$$(4.13) \qquad v = c_o H_2^{(1)}(kr) \sin 2\theta + c_2 H_{2/3}^{(1)}(kr) \sin \frac{2\theta}{3} .$$

When kr is large (4.13) becomes

$$(4.14) \qquad v = c_o \sqrt{\frac{2}{\pi kr}} e^{i(kr - \frac{5\pi}{4})} \sin 2\theta + c_2 \sqrt{\frac{2}{\pi kr}} e^{i(kr - \frac{7\pi}{12})} \sin \frac{2\theta}{3} .$$

We expect the far field of the z-component of the magnetic field vector u to have a similar form, except that it contains surface wave terms, which are

appreciable only near the wedge surfaces.

Hence

$$(4.15) \qquad u = \left[\frac{m(\theta)}{\sqrt{r}}\, e^{ikr} + O\left(\frac{1}{r^{3/2}}\right)\right] + \text{surface waves}$$

where $m(\theta)$ is unknown. If we substitute (4.6), (4.7), (4.14) and (4.15) into (4.12), and keep terms of order $r^{-1/2}$, we obtain

$$(4.16) \quad m(\theta) = \sqrt{\frac{2}{\pi k}}\, \frac{\left[c_o e^{-i\frac{5\pi}{4}} \sin 2\theta + c_2 e^{-i\frac{7\pi}{12}} \sin \frac{2\theta}{3}\right]}{(ik)^2 + ik\lambda(\cos\theta - \sin\theta) - \lambda^2} .$$

When $\frac{\lambda}{k}$ is small the far field is given by

$$(4.17) \qquad u = \sqrt{\frac{2\pi}{kr}}\, \left(\frac{1+i\sqrt{3}}{4}\right) e^{i(kr - \frac{\pi}{12})} \sin 2\theta ,$$

and when $\frac{\lambda}{k}$ is large the far field is given by

$$(4.18) \qquad u = \sqrt{\frac{2\pi}{kr}}\, \left(\frac{\sqrt{2}}{1+i\sqrt{3}}\right) \left(\frac{k}{\lambda}\right)^{2/3} e^{i(kr - \frac{\pi}{12})} \sin \frac{2\theta}{3} .$$

It is worth noting that (4.16) shows that $m(\theta)=0$ when $\theta=0$ or when $\theta = \frac{3\pi}{2}$. Hence, from (4.15) the radiated field of u is $O(r^{-3/2})$ on the wedge surfaces, so that the surface wave terms soon predominate. The fact that the radiated field is $O(r^{-3/2})$ at the wedge surfaces is a natural consequence of the boundary condition and is independent of the wedge angle. Indeed, at the horizontal and vertical wedge surfaces, $\frac{\partial u}{\partial n} = \pm \frac{1}{r} \frac{\partial u}{\partial \theta}$, and then (4.15) shows that $\frac{\partial u}{\partial n}$ vanishes more rapidly than does u. Hence, in order to fulfill the boundary condition, the function $m(\theta)$ must vanish on the wedge.

Appendix I

Calculation of the integrals H and J

The integrals H and J are defined as follows:

$$H = \int_0^\infty \int_{\frac{\pi}{2}}^{0} e^{-\lambda r(\cos\theta - \sin\theta)} H_1^{(1)}(kr)\cos\theta\, r\,dr\,d\theta - \int_0^\infty \int_{\frac{3\pi}{2}}^{2\pi} e^{-\lambda r(\cos\theta-\sin\theta)} H_1^{(1)}(kr)\cos\theta\, r\,dr\,d\theta$$

$$J = \int_0^\infty \int_{-\frac{\pi}{2}}^{0} e^{-\lambda r(\cos\theta-\sin\theta)} H_{2/3}^{(1)}(kr)\sin\frac{2\theta}{3}\, r\,dr\,d\theta - \int_0^\infty \int_{\frac{3\pi}{2}}^{2\pi} e^{-\lambda r(\cos\theta-\sin\theta)} H_{2/3}^{(1)}(kr)\sin\frac{2\theta}{3}\, r\,dr\,d\theta$$

Let us first consider H. By changing the range of integration of the θ-variable in both integrals to the first quadrant we obtain

$$H = \int_0^\infty \int_0^{\frac{\pi}{2}} e^{-\lambda r(\cos\theta+\sin\theta)} H_1^{(1)}(kr)\sin\theta\, r\,dr\,d\theta - \int_0^\infty \int_0^{\frac{\pi}{2}} e^{-\lambda r(\cos\theta+\sin\theta)} H_1^{(1)}(kr)\sin\theta\, r\,dr\,d\theta$$

or

$$H = 0.$$

We next consider J. By changing the range of integration in both integrals as before, and combining, we obtain

$$J = \sqrt{3} \int_0^\infty \int_0^{\frac{\pi}{2}} e^{-\lambda r(\cos\theta + \sin\theta)} H_{2/3}^{(1)}(kr)\cos\frac{2}{3}(\theta - \pi)\, r\,dr\,d\theta .$$

By making the change of variable $\psi = \theta - \frac{\pi}{4}$, the above equation becomes

$$J = \sqrt{3} \int_0^\infty H_{2/3}^{(1)}(kr) \left\{ \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-\lambda\sqrt{2}\, r\cos\psi} \sin\frac{2}{3}\psi\, d\psi \right\} r\,dr .$$

But the integrand of the above is an odd function of ψ and hence

$$J = 0.$$

Appendix II

Limiting expressions for I_ν

We wish to determine limiting expressions for the following integral for small and large values of $\frac{\lambda}{k}$:

$$I_\nu = \int_0^\infty e^{-\lambda t} H_\nu^{(1)}(kt)dt .$$

Suppose that $\frac{\lambda}{k}$ is small. Using (3.16) we can write I_ν in the form

$$I_\nu = \frac{\left[\sqrt{(\frac{\lambda}{k})^2+1} - (\frac{\lambda}{k})\right]^\nu}{k\sqrt{(\frac{\lambda}{k})^2+1}} \left\langle 1 + \frac{i}{\sin \nu\pi}\left\{\cos \nu\pi - \left[(\frac{\lambda}{k}) + \sqrt{(\frac{\lambda}{k})^2+1}\right]^{2\nu}\right\}\right\rangle .$$

Now for small values of $\frac{\lambda}{k}$ we have

$$\sqrt{(\frac{\lambda}{k})^2+1} \approx 1$$

$$\sqrt{(\frac{\lambda}{k})^2+1} \pm (\frac{\lambda}{k}) \approx 1.$$

Hence

$$I_\nu = \frac{1}{k \sin \nu\pi}\left[e^{i\nu\pi} + e^{-i\frac{\pi}{2}}\right] .$$

When $\nu = \frac{1}{3}$,

$$I_{1/3} = \frac{1}{k}\left[1 - i\frac{1}{\sqrt{3}}\right]$$

and when $\nu = \frac{2}{3}$,

$$I_{2/3} = \frac{1}{k}\left[1 - i\sqrt{3}\right] .$$

Suppose next that $\frac{\lambda}{k}$ is large. We can write I_ν in the form

$$I_\nu = \frac{(\sqrt{\lambda^2+k^2}-\lambda)^\nu}{k^\nu \sqrt{\lambda^2+k^2}} \left\{ 1 + \frac{i}{\sin \nu\pi} \left[\cos \nu\pi - \frac{(\lambda+\sqrt{k^2+\lambda^2})^{2\nu}}{k^{2\nu}} \right] \right\} .$$

Now

$$\sqrt{\lambda^2 + k^2} = \lambda \sqrt{1 + (\frac{k}{\lambda})^2} \approx \lambda$$

$$\sqrt{\lambda^2 + k^2} - \lambda = \lambda \left\{ \sqrt{1 + (\frac{k}{\lambda})^2} - 1 \right\} \approx 0$$

$$\sqrt{\lambda^2 + k^2} + \lambda = \lambda \left\{ \sqrt{1 + (\frac{k}{\lambda})^2} + 1 \right\} \approx 2\lambda$$

and the expression for I_ν reduces to

$$I_\nu = \frac{-i}{\sin \nu\pi} \frac{1}{\lambda} (\frac{2\lambda}{k})^\nu .$$

When $\nu = 1/3$

$$I_{1/3} = -\frac{i2}{\sqrt{3}} (\frac{2}{k})^{1/3} \frac{1}{\lambda^{2/3}} ,$$

and when $\nu = 2/3$

$$I_{2/3} = -\frac{2}{\sqrt{3}} (\frac{2}{k})^{2/3} \frac{1}{\lambda^{1/3}} .$$

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